5 Prove that

a $(\sin x + \cos x)^2 \equiv 1 + 2\sin x \cos x$

b $\dfrac{1}{\cos x} - \cos x \equiv \sin x \tan x, \quad \cos x \neq 0$

c $\dfrac{\cos^2 x}{1 - \sin x} \equiv 1 + \sin x, \quad \sin x \neq 1$

d $\dfrac{1 + \sin x}{\cos x} \equiv \dfrac{\cos x}{1 - \sin x}, \quad \cos x \neq 0$

6 **a** Prove the identity

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 \equiv 2 + \tan^2 x.$$

b Hence find, in terms of π, the values of x in the interval $0 \leq x \leq 2\pi$ such that

$$(\cos x - \tan x)^2 + (\sin x + 1)^2 = 3.$$

7

$$f(x) \equiv \cos^2 x + 2\sin x, \quad 0 \leq x \leq 2\pi.$$

a Prove that f(x) can be expressed in the form

$$f(x) = 2 - (\sin x - 1)^2.$$

b Hence deduce the maximum value of f(x) and the value of x for which this occurs.

Simplify each trig expression. (6 pts)

1) $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$

$\frac{\cos^2 x}{(1+\sin x)(\cos x)} + \frac{(1+\sin x)(1+\sin x)}{(1+\sin x)(\cos x)}$

$= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)(\cos x)}$

$= \frac{1 + 1 + 2\sin}{\cos x(1+\sin x)}$

$= \frac{2 + 2\sin x}{\cos x(1+\sin x)}$

$= \frac{2(1+\sin x)}{\cos x(1+\sin x)}$

$= \frac{2}{\cos x}$

2) sinx – sinx cos²x

$= \sin x - \sin x(1-\sin^2 x)$

$= \sin x - \sin x + \sin^3 x$

$= \sin^3 x$

No limits

Simplify each of the following expressions:

a $1 - \cos^2 \frac{1}{2}\theta$

b $5 \sin^2 3\theta + 5 \cos^2 3\theta$

c $\sin^2 A - 1$

d $\dfrac{\sin \theta}{\tan \theta}$

e $\dfrac{\sqrt{1 - \cos^2 x}}{\cos x}$

f $\dfrac{\sqrt{1 - \cos^2 3A}}{\sqrt{1 - \sin^2 3A}}$

g $(1 + \sin x)^2 + (1 - \sin x)^2 + 2 \cos^2 x$

h $\sin^4 \theta + \sin^2 \theta \cos^2 \theta$

i $\sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta + \cos^4 \theta$

56. For x such that $0 < x < \frac{\pi}{2}$, the expression

$$\frac{\sqrt{1-\cos^2 x}}{\sin x} + \frac{\sqrt{1-\sin^2 x}}{\cos x}$$ is equivalent to:

F. 0
G. 1
H. 2
J. $-\tan x$
K. $\sin 2x$

50. If $\cos\theta = \frac{6}{17}$ and $\tan\theta = \frac{5}{6}$, then $\sin\theta = ?$

f. $\frac{5}{17}$

g. $\frac{6}{5}$

h. $\frac{17}{5}$

i. $\frac{5}{6}$

j. $\frac{1}{2}$

60. Whenever $\frac{2\cos\alpha\sin\alpha}{\cos^2\alpha + 1 - \sin^2\alpha}$ is defined, it simplifies to:

DO YOUR FIGURING HERE.

F. $\tan\alpha$

G. $\cot\alpha$

H. 2

J. $\frac{2}{\cos\alpha - \sin\alpha}$

K. $\sin\alpha\ \cos\alpha$

Becomes

$$\frac{2\cos\alpha \sin\alpha}{\cos^2\alpha + \cos^2 x} =$$

$$\frac{2\cos\alpha \sin\alpha}{2\cos^2 x} =$$

END OF TEST 2

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.

DO NOT RETURN TO THE PREVIOUS TEST.

$$\frac{\cos\alpha \sin\alpha}{\cos\alpha \cos\alpha} =$$

$$\frac{\sin\alpha}{} = \tan\alpha$$

128 − 120 = ? = 8

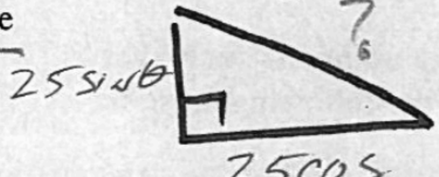
Trigonometry

58. A right triangle has legs of length 25 sin θ feet and 25 cos θ feet for some angle θ that satisfies 0° < θ < 90°. What is the length, in feet, of the longest side of the triangle?

F. 0
G. 1
H. 5
J. 25
K. Cannot be determined from the given information

J

Longest side of right triangle is the hypotenuse



$(25\sin\theta)^2 + (25\cos\theta)^2 = (?)^2$

$625(\sin^2\theta) + 625(\cos^2\theta) = ?^2$

$625(\sin^2\theta + \cos^2\theta) = (?)^2$

$625(1) = (?)^2$

$\sqrt{625} = \sqrt{?^2}$

$25 = ?$

59. For all nonzero x, y, and z such that $x = yz$, which of the following *must* be equivalent to xy ?

A. $\frac{z}{x}$

B. yz^2

$x = x$

Solving for variables using substitution

Example Question #2 : How To Find The Tangent Of An Angle

Consider a right triangle with an inner angle x $(x < 90^\circ)$ **.**

If

$\cos x = \frac{3}{5}$

and

$\sin x = \frac{4}{5}$

what is $\tan x$ **?**

Possible Answers:

$\frac{1}{5}$

5

$\frac{4}{3}$

1

$\frac{3}{4}$

Correct answer:

$\frac{4}{3}$

Explanation:

The tangent of an angle *x* is defined as

$\tan x = \frac{\sin x}{\cos x}$

Substituting the given values for cos *x* and sin *x*, we get

$\frac{4/5}{3/5} = 4/3$

If the sine of an angle equals $2/3$, and the cosine of the same angle equals $5/12$, what is the tangent of the angle?

Possible Answers:

$8/5$

$5/8$

$12/5$

$12/8$

$8/12$

Correct answer:

$8/5$

Explanation:

$$Sine = \frac{opposite}{hypotenuse}.\ Cosine = \frac{adjacent}{hypotenuse}.\ Tangent = \frac{opposite}{adjacent}$$

The cosine of the angle is $5/12$ and since that is a reduced fraction, we know the hypotenuse is 12 and the adjacent side equals 5.

The sine of the angle equals $2/3$, and since the hyptenuse is already 12 we know that we must multiply the numerator and denominator by 4 to get t
common denominator of 12. Therefore, the opposite side equals 8.

Since $tangent = \frac{opposite}{}$, , the answer is $8/5$.

Simplify $(\cos\Theta - \sin\Theta)^2$

Possible Answers:

$1 + \cos 2\Theta$

$1 + \sin 2\Theta$

$\sin 2\Theta - 1$

$1 - \sin 2\Theta$

$\cos 2\Theta - 1$

Correct answer:

$1 - \sin 2\Theta$

Explanation:

Multiply out the quadratic equation to get $\cos\Theta^2 - 2\cos\Theta\sin\Theta + \sin\Theta^2$

Then use the following trig identities to simplify the expression:

$\sin 2\Theta = 2\sin\Theta\cos\Theta$

$\sin\Theta^2 + \cos\Theta^2 = 1$

$1 - \sin 2\Theta$ is the correct answer for $(\cos\Theta - \sin\Theta)^2$

$1 + \sin 2\Theta$ is the correct answer for $(\cos\Theta + \sin\Theta)^2$

Which of the following is equal to $cos\ x * csc\ x$?

Possible Answers:

$cot\ x * sec\ x$

$cot\ x$

$tan\ x$

$sin\ x * sec\ x$

$sec\ x$

Correct answer:

$cot\ x$

Explanation:

Here, we use the SOHCAHTOA ratios and the fact that csc x = 1 / sin x.

Cosine x = adjacent side length / hypotenuse length

Cosecant x = 1 / sin x = hypotenuse / opposite

(Adjacent / hypotenuse) * (hypotenuse / opposite) = Adjacent / opposite = Cotangent x.

Simplify: $(\sin\Theta + \cos\Theta)^2$

Possible Answers:

$\cos 2\Theta - 1$

$1 + \cos 2\Theta$

$1 + \sin 2\Theta$

None of the answers are correct

$2\sin\Theta\cos\Theta - 1$

Correct answer:

$1 + \sin 2\Theta$

Explanation:

Using the foil method, multiply. Simplify using the Pythagorean identity $\sin^2\Theta + \cos^2\Theta = 1$ and the double angle identity $\sin 2\Theta = 2\sin\Theta\cos\Theta$.

Which of the following is equivalent to cot(θ)sec(θ)sin(θ)

Possible Answers:

0

1

cot(θ)

tan(θ)

−sec(θ)

Correct answer:

1

Explanation:

The first thing to do is to breakdown the meaning of each trig function, cot = cos/sin, sec = 1/cos, and sin = sin. Then put these back into the function and simplify if possible, so then (cos (Θ)/sin (Θ))*(1/cos (Θ))*(sin (Θ)) = (cos (Θ)*sin(Θ))/(sin (Θ)*cos(Θ)) = 1, since they all cancel out.

Using trigonometry identities, simplify $\sin\theta\cos^2\theta - \sin\theta$

Possible Answers:

$\cos^2\theta\sin\theta$

$\cos^3\theta$

None of these answers are correct

$-\sin^3\theta$

$\sin^2\theta\cos\theta$

Correct answer:

$-\sin^3\theta$

Explanation:

Factor the expression to get $\sin\theta(\cos^2\theta - 1)$.

The trig identity $\cos^2\theta + \sin^2\theta = 1$ can be reworked to becomes $\cos^2\theta - 1 = -\sin\theta$ resulting in the simplification of $-\sin^3\theta$.

Using trig identities, simplify sinθ + cotθcosθ

Possible Answers:

tanθ

$\cos^2\theta$

cscθ

secθ

$\sin^2\theta$

Correct answer:

cscθ

Explanation:

Cotθ can be written as cosθ/sinθ, which results in sinθ + $\cos^2\theta$/sinθ.

Combining to get a single fraction results in $(\sin^2\theta + \cos^2\theta)/\sin\theta$.

Knowing that $\sin^2\theta + \cos^2\theta = 1$, we get 1/sinθ, which can be written as cscθ.

Simplify $\sec^4\Theta - \tan^4\Theta$.

Possible Answers:

$\sec^2\Theta + \tan^2\Theta$

$\sec\Theta + \sin\Theta$

$\cos\Theta - \sin\Theta$

$\tan^2\Theta - \sin^2\Theta$

$\sin\Theta + \cos\Theta$

Correct answer:

$\sec^2\Theta + \tan^2\Theta$

Explanation:

Factor using the difference of two squares: $a^2 - b^2 = (a+b)(a-b)$

The identity $1 + \tan^2\Theta = \sec^2\Theta$ which can be rewritten as $1 = \sec^2\Theta - \tan^2\Theta$

So $\sec^4\Theta - \tan^4\Theta = (\sec^2\Theta + \tan^2\Theta)(\sec^2\Theta - \tan^2\Theta) = (\sec^2\Theta + \tan^2\Theta)(1) = \sec^2\Theta + \tan^2\Theta$

Simplify the following expression:

$$\frac{sin\theta}{cot\theta sec\theta}$$

Possible Answers:

cscΘ

$\cos^2\Theta$

$\sin^2\Theta$

None of the answers are correct

tanΘ

Correct answer:

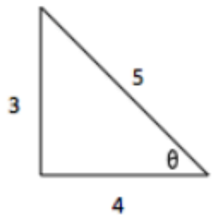
$\sin^2\Theta$

Explanation:

Convert cotΘ and secΘ to sinΘ and cosΘ and simplify the resulting complex fraction.

$\cot\Theta = \frac{\cos\Theta}{\sin\Theta}$ $\quad \sec\Theta = \frac{1}{\cos\Theta}$

Given this right triangle, what is the value of $\frac{sin\theta}{cos\theta}$?



Possible Answers:

$\frac{3}{4}$

$\frac{3}{5}$

$\frac{4}{5}$

$\frac{4}{3}$

Correct answer:

$\frac{3}{4}$

Explanation:

To solve this question, you must know SOHCAHTOA. Sin divided by cosine is the tangent of the angle. Tan = opposite / adjacent = 3/4.

If the sine of an angle equals $2/3$, and the cosine of the same angle equals $5/12$, what is the tangent of the angle?

Possible Answers:

8/5

5/8

12/5

12/8

8/12

Correct answer:

8/5

Explanation:

$$Sine = \frac{opposite}{hypotenuse}.\ Cosine = \frac{adjacent}{hypotenuse}.\ Tangent = \frac{opposite}{adjacent}$$

The cosine of the angle is $5/12$ and since that is a reduced fraction, we know the hypotenuse is 12 and the adjacent side equals 5.

The sine of the angle equals $2/3$, and since the hyptenuse is already 12 we know that we must multiply the numerator and denominator by 4 to get the common denominator of 12. Therefore, the opposite side equals 8.

Since $tangent = \frac{opposite}{adjacent}$,, the answer is $8/5$.